

# A PHASE-SPACE DISCONTINUOUS GALERKIN APPROXIMATION FOR THE RADIATIVE TRANSFER EQUATION IN SLAB GEOMETRY

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In this work we consider the numerical solution of the second-order form of the radiative transfer equation (RTE) in slab geometry, employing a SIP discontinuous Galerkin method and several error estimators for adaptivity of the mesh.

Due to the product structure of the phase-space in this formulation, classical numerical schemes for the RTE, such as the  $P_N$ -approximations, use separate discretization techniques for the spatial and angular variables. However, a major drawback of the independent discretizations is that a local refinement in phase-space is not possible. Therefore, the aim is to develop a numerical method for the approximation of the RTE that allows for local mesh refinement in phase-space and that allows for a relatively simple analysis and implementation [1]. To accomplish this, we employ quad-tree grids as partitions for the phase-space and a symmetric interior penalty discontinuous Galerkin formulation for the discrete problem.

Besides the proper treatment of traces, which requires the inclusion of a weight function in our case, the analysis of the overall scheme is along the standard steps for the analysis of discontinuous Galerkin methods. Supporting examples show the accuracy and stability of the method also numerically for different polynomial degrees.

For the local adaptation of the grid we investigate several error estimators. First, we consider two hierarchical error estimators, which use polynomials of higher degree, and the discrete solution on a uniformly refined mesh, respectively. To overcome the fact that these estimators require the solution of an additional global problem in every step, we then propose an a posteriori estimator based on a local averaging procedure, which shows performances comparable to the more costly hierarchical estimators.

[1] - Bardin, R., Bertrand, F., Palii, O. and Schlottbom, M. - A phase-space discontinuous Galerkin approximation for the radiative transfer equation in slab geometry, arXiv:2201.06104v3 (2023)

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