Approximation of Functionals by Neural Network without Curse of Dimensionality

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Recently, many methods have been developed for solving partial differential equations (PDEs) by neural networks. However, the curse of dimensionality (CoD) is a serious issue that generally exists in this field when dealing with high dimensional problems. In this work, we establish a new method for the approximation of functionals by neural networks without CoD by defining (i) a Fourier-type series on the infinite-dimensional space of functionals and (ii) the associated Barron spectral space $\mathcal{B}_s$ and a Hilbert space $\mathcal{H}_s$ of functionals. The approximation error of the designed neural network in this method is $O(1/\sqrt{m})$ where m is the size of networks. Then, the proposed method is employed in several numerical experiments, such as evaluating the energy functionals, solving two-dimensional and four-dimensional Poisson equations by aforementioned neural networks at one or a few given points.

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