AFEM for the fractional Laplacian

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For the discretization of the integral fractional Laplacian $(-\Delta)^s$, $0 < s < 1$, based on piecewise linear functions, we present and analyze a reliable weighted residual a posteriori error estimator. In order to compensate for a lack of L^2-regularity of the residual in the regime $3/4 < s < 1$, this weighted residual error estimator includes as an additional weight a power of the distance from the mesh skeleton. We prove optimal convergence rates for an h-adaptive algorithm driven by this error estimator in the framework of [Carstensen, Feischl, Page, Praetorius, Axioms of adaptivity, CAMWA (2018)] . Key to the analysis of the adaptive algorithm are local inverse estimates for the fractional Laplacian. These local inverse estimates have further applications. For example, they underlie the proof that multilevel diagonal scaling leads to uniformly bounded condition numbers even in the presence of locally refined meshes. In the second part of the talk, we will present one such optimal multilevel diagonal scaling preconditioner.

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