Inf-Sup Theory for the Biot equations. Part 2: Discretization

Pietro Zanotti
Università degli Studi di Pavia, Italy
pietro.zanotti@unipv.it

The quasi-static Biot equations in poroelasticity describe the flow of a Newtonian fluid inside an elastic porous medium. We propose a new finite element discretization of the equations, based on the inf-sup theory presented in another talk by Christian Kreuzer.

The discretization involves the two main unknowns of the equations, namely the displacement of the elastic medium and pressure of the fluid, as well as the total pressure and the total fluid content, two auxiliary variables playing a central role in our analysis. We make use of the backward Euler scheme in time and approximate all variables in space by conforming Lagrange finite elements on simplicial meshes.

We establish the well-posedness, the stability and the quasi-optimality of the discretization. All the constants involved in our analysis are robust with respect to the material parameters. We additionally discuss the preconditioning of the linear system to be solved at each time step.

Joint work with Christian Kreuzer (TU Dortmund).