Adaptive FEM for linear elliptic PDEs: Optimal complexity

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We consider a general nonsymmetric second-order linear elliptic PDE in the framework of the Lax–Milgram lemma. We formulate and analyze an AFEM algorithm that steers the adaptive mesh-refinement and the inexact iterative solution of the arising linear systems. More precisely, the iterative solver employs, as an outer loop, the so-called Zarantonello iteration to symmetrize the system and, as an inner loop, a uniformly contractive algebraic solver, e.g., an optimally preconditioned conjugate gradient method or an optimal geometric multigrid algorithm. We show that the proposed inexact adaptive iteratively symmetrized finite element method (AISFEM) leads to full linear convergence and, for sufficiently small adaptivity parameters, to optimal convergence rates with respect to the overall computational cost, i.e., the total computational time.

The talk is based on our recent preprints "Adaptive FEM with quasi-optimal overall cost for nonsymmetric linear elliptic PDEs" (arXiv:2212.00353) and "hp-robust multigrid solver on locally refined meshes for FEM discretizations of symmetric elliptic PDEs" (arXiv:2210.10415).

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