Algebraic Geometry codes in the sum-rank metric

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Linear codes in the Hamming metric have been playing a central role in the theory of error correction since the 50s and were extensively studied. In reverse, the theory of codes in the sum-rank metric is still in its beginnings, and to date, only a few constructions are known.

Algebraic Geometry codes in the Hamming metric allow overcoming the main drawback of Reed-Solomon codes, which is that their length is bounded by the cardinality of the finite field we work on, while benefiting from good parameters. The counterpart of Reed-Solomon codes in the sum-rank metric are linearized Reed-Solomon codes. They have optimal parameters but suffer from the same limitation as Reed-Solomon codes. However, in contrast with the situation of codes in the Hamming metric, no geometric construction has been proposed so far.

In this talk, we will present the first geometric construction of sum-rank metric codes, called linearized Algebraic Geometry codes. After introducing some background on codes in the sum-rank metric, we will develop the theory of Riemann-Roch spaces over Ore polynomials rings with coefficients in the function field of a curve, by exploiting the classical theory of divisors and Riemann-Roch spaces on algebraic curves. With this theory at hand, we will study the parameters of linearized Algebraic Geometry codes and give lower bounds for their dimension and minimum distance. Notably, we will show that our new codes exhibit quite good parameters, respecting a similar bound to Goppa's bound for Algebraic Geometry codes in the Hamming metric.

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