Applications and Constructions of Trisections of Low Genus on Elliptic Surfaces

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Consider a rational elliptic surface over a field k with char 0 given by $\mathcal{E} : y^2 = x^3 + f(t)x + g(t)$, with $f, g \in k[t]$, $\deg(f) \leq 4$ and $\deg(g) \leq 6$. If all the bad fibres are irreducible, such a surface comes from the blow-up of a del Pezzo surface of degree 1. We are interested in studying multisections, i.e. curves which intersect each fibre a fixed number of times; more precisely trisections (three times). Many configurations of singularities on a trisection lead to a lower genus; here we focus on one of them. By specifying conditions on the coefficients f, g of the surface $\mathcal{E}$, and looking at trisections which pass through a given point three times, we obtain a pencil of cubics on such surfaces. Our construction could have interesting applications in proving the Zariski density of the rational points. It is especially interesting since the results in this regard are partial for del Pezzo surfaces of degree 1. Further reduction of the genus of a trisection with a triple singularity is possible, however this process leads to an isolated curve of genus zero instead of a family.

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