On percolation in locally dependent random graphs

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Consider a random subgraph of the square integer lattice $\mathbb{Z}^2$ obtained by including each edge independently at random with probability p, and omitting it otherwise. The Harris–Kesten theorem states that if $p \leq 1/2$, then almost surely all connected components in this random graph model are finite, while if $p > 1/2$ then almost surely the model percolates and there exists a unique infinite connected component.

But now what if we introduced some local dependencies between the edges? More precisely, suppose each edge still has a probability p of being included in our random subgraph, but its state (present/absent) may depend on the states of nearby edges. To what extent can we exploit such local dependencies to delay the emergence of an infinite component?

In this talk I will discuss this question, which first arose in work of Balister, Bollobás and Walters in 2005, as well as some recent progress around it.

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