Complete non-ambiguous trees and associated permutations: connections through the Abelian sandpile model

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Complete non-ambiguous trees (CNATs) were originally introduced by Aval et al. (2014) as a special case of tree-like tableaux. We can associate a permutation to a CNAT by keeping only its leaf dots. In recent work, Chen and Ohlig (2022) initiated the first in-depth combinatorial study of this relationship, notably showing that the number of n-permutations that are associated with exactly one CNAT is 2^{n-2}. We extend this work by enumerating permutations associated with exactly k CNATs for various values of $k > 1$, via bijective approaches. Our results rely on a connection to the so-called Abelian sandpile model (see Dukes et al., 2019). We also exhibit a new bijection between $(n-1)$-permutations and CNATs whose permutation is the decreasing permutation $n(n-1)\cdots 1$. This bijection maps the left-to-right minima of the permutation to dots on the bottom row of the corresponding CNAT, and descents of the permutation to empty rows of the CNAT.

References

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