Common linear patterns are rare

Nina Kamčev University of Zagreb, Croatia nina.kamcev@math.hr

Several classical results in Ramsey theory (including famous theorems of Schur, van der Waerden, Rado) deal with finding monochromatic linear patterns in two-colourings of the integers. Our topic will be quantitative extensions of such results. A linear system L over \mathbb{F}_q is common if the number of monochromatic solutions to L = 0 in any two-colouring of \mathbb{F}_q^n is asymptotically at least the expected number of monochromatic solutions in a random two-colouring of \mathbb{F}_q^n . Motivated by existing results for specific systems (such as Schur triples and arithmetic progressions), as well as extensive research on common and Sidorenko graphs, the systematic study of common systems of linear equations was recently initiated by Saad and Wolf. Fox, Pham and Zhao characterised common linear equations.

I will talk about recent progress towards a classification of common systems of two or more linear equations. In particular, any system containing a four-term arithmetic progression is uncommon. This follows from a more general result which allows us to deduce the uncommonness of a general system from certain properties of one- or two-equation subsystems.

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