Extending Morse-Forman theory to vector functions

Guillaume Brouillette
Université de Sherbrooke, Canada
Guillaume.Brouillette2@USherbrooke.ca

In the last decades, discrete Morse theory, or Morse-Forman theory, has proven itself to be useful for a wide range of applications, notably in topological data analysis. In particular, when analyzing a simplicial complex filtered by a real-valued function, the theory may be used to reduce its complexity and to optimize the computation of its associated persistent homology. Recent work has shown that it can be adapted in order to compute more efficiently the multipersistent homology of a complex induced by a vector function. Nevertheless, the theorical implications and geometrical insights of this adaptation have not yet been extensively investigated.

To gain some perspective on the matter, in this presentation, we extend Morse-Forman theory to vector-valued functions, thus defining the concept of multidimensional discrete Morse (MDM) functions. To do so, we use notions of combinatorial dynamics as defined in recent years to adapt the main definitions and theorems of Forman to the vectorial setting. This leads to a more general result than that of Forman concerning the sublevel sets of a MDM function and to an alternative approach on the matter. Moreover, we propose a way to derive a Morse decomposition in critical components from the critical points of a MDM function. This is specific to the multidimensional setting since critical points of real discrete Morse functions are isolated, and thus do not form components. Finally, from this Morse decomposition, we deduce new Morse equation and inequalities.

Joint work with Madjid Allili (Bishop's University, Canada) and Tomasz Kaczynski (Université de Sherbrooke, Canada).