Discrete group actions on 3-manifolds and embeddable Cayley complexes

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A theorem of Maschke [2, p. 287] states that a finite group acts discretely and topologically on $\mathbf{S}^2$ if and only if it has an alternative Cayley graph that embeds equivariantly in $\mathbf{S}^2$. Recently, Georgakopoulos [1] generalised this theorem to finitely generated groups. We extend the above results to three dimensions. Namely, we prove that a finitely generated group Γ admits a discrete topological action on a simply connected 3-manifold if and only if Γ has a generalised Cayley complex that embeds equivariantly in one of the following four 3-manifolds: (i) $\mathbf{S}^3$, (ii) $\mathbf{R}^3$, (iii) $\mathbf{S}^2 \times \mathbf{R}$, and (iv) the complement of a tame Cantor set in $\mathbf{S}^3$. In the process, we derive a combinatorial characterization of the finitely generated groups that act discretely and topologically on simply connected 3-manifolds.

[1] Georgakopoulos, A. On planar Cayley graphs and Kleinian groups. Trans. Amer. Math. Soc. Vol. 373, pp. 4649-4684, 2020.

[2] Gross, J.L. and Tucker, T.W. (1987). Topological Graph Theory. John Wiley & Sons.

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