The Gromov-Hausdorff distance between spheres

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The Gromov-Hausdorff distance is a fundamental tool in Riemannian geometry (through the topology it generates) and is also utilized in Applied Geometry and Topological Data Analysis as a metric for expressing the stability of methods which process geometric data (e.g. hierarchical clustering and persistent homology barcodes via the Vietoris-Rips filtration). In fact, distances such as the Gromov-Hausdorff distance or its Optimal Transport variants (i.e. the so-called Gromov-Wasserstein distances) are nowadays often invoked in Machine Learning Applications.

Whereas it is often easy to estimate the value of the Gromov-Hausdorff distance between two given metric spaces, its precise value is rarely easy to determine. Some of the best estimates follow from considerations related to both the stability of persistent homology features and to Gromov's filling radius. However, these turn out to be non-sharp.

In this talk, I will describe these estimates and also results which permit calculating the precise value of the Gromov-Hausdorff between pairs of spheres (endowed with their usual geodesic distance). These results involve lower bounds which arise from a certain version of the Borsuk-Ulam theorem that is applicable to discontinuous maps, and also matching upper bounds which are induced from specialized constructions of (a posteriori optimal) "correspondences" between spheres.