

ERROR ESTIMATES OF NUMERICAL METHODS FOR THE LONG-TIME DYNAMICS OF THE NONLINEAR KLEIN-GORDON EQUATION

Yue Feng

Sorbonne University, France
yue.feng@sorbonne-universite.fr

I will present the error estimates of numerical methods for the long-time dynamics of the nonlinear Klein-Gordon equation (NKGE) with weak nonlinearity, which is characterized by ε^2 with $\varepsilon \in (0, 1]$ a dimensionless parameter. Different numerical methods are adopted to discretize the NKGE and rigorous error bounds are established for the long-time dynamics. Numerical methods include finite difference methods, exponential wave integrators and time-splitting methods with particular attentions paid on error bounds of different numerical methods explicitly depending on the mesh size h , time step τ as well as the parameter ε up to the time $t = T/\varepsilon^2$ with $T > 0$ fixed. As a by-product, our results are extended to an oscillatory NKGE whose solution propagates waves with wavelength at $O(1)$ in space and $O(\varepsilon^2)$ in time. Extensive numerical examples are provided to confirm our error bounds and demonstrate that they are sharp.

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