A factorization approach to the extrinsic approximation of volume-preserving diffeomorphisms

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We present a semi-Lagrangian numerical method for diffeomorphism approximation and its application to incompressible hydrodynamics on the sphere. The method approximates the flow of a velocity field using a spatio-temporal discretization formed by a composition of submaps. This technique substitutes the effects of spatial refinement with the operation of composition by adaptively growing the temporal discretization. In turn, the method has the capacity of accurately and sparsely representing the generation of fine scales globally using only a linear increase in the degrees of freedom. Based on a factorization of diffeomorphisms developed by Modin, we design a geometric correction technique to constrain the evolution to a tubular neighbourhood of the volume-preserving diffeomorphism group. An analysis on the adaptive use of this correction technique in conjunction with the submap decomposition is given and supported through numerical experimentation on some canonical geophysical flows. We demonstrate the ability to improve the conservation properties of the method and capture turbulent energy cascades at subgrid scales.

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