Curvature approximation in arbitrary dimension with Regge finite elements

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We study the curvature of Regge metrics on simplicial triangulations of dimension N. Here, a Riemannian metric is called a Regge metric if it is piecewise smooth and its tangential-tangential components are single-valued on every codimension-1 simplex in the triangulation. When such a metric is piecewise polynomial, it belongs to a finite element space called the Regge finite element space. Regge metrics are not classically differentiable, but it turns out that one can still make sense of their curvature in a distributional sense. In the lowest-order setting, the distributional curvature of a Regge metric is a linear combination of Dirac delta distributions supported on codimension-2 simplices S, weighted by the angle at S: 2π minus the sum of the dihedral angles incident at S. For piecewise polynomial Regge metrics of higher degree, the distributional curvature includes additional contributions involving the curvature in the interior of each N-simplex and the jump in the mean curvature across each codimension-1 simplex.

We study the convergence of the distributional curvature under refinement of the triangulation. We show that in the H^{-2}-norm, this convergence takes place at a rate of $O(h^{r+1})$ when a smooth Riemannian metric is interpolated by a piecewise polynomial Regge metric of degree $r \geq 0$ on a triangulation whose maximum simplex diameter is h, provided that either $N = 2$ or $r \geq 1$.

Joint work with Yakov Berchenko-Kogan (Florida Institute of Technology, USA) and Michael Neunteufel (TU Wien, Austria).