

ON THE CONTRACTIVITY OF ODES AND NUMERICAL INTEGRATORS ON RIEMANNIAN  
MANIFOLDS

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On the contractivity of ODEs and numerical integrators on Riemannian manifolds

In the theory of differential equations concepts related to stability are very well established and understood. Parallel theories for numerical schemes in Euclidean spaces are also well developed. From the beginning, the notion of absolute stability, using a linear test equation was the prevalent tool. Later, Dahlquist, Burrage and Butcher were leading the development of a numerical stability theory that also makes sense for nonlinear problems, that of B-stable or contractive methods, set in Hilbert spaces. In more recent times, it has become popular to construct and analyse numerical schemes for differentiable manifolds, methods that are entirely intrinsic, examples are the Lie group integrators. For the study of nonlinear stability, Riemannian manifolds, or even Finsler manifolds, seem to be a useful framework for studying contractive methods. Building on work by Kunzinger et al. [1] and Simpson-Porco & Bullo [2] we shall suggest a way to generalise the notion of B-stability to Riemannian manifolds. We study the geodesic implicit Lie-Euler method as a model method and show some first theoretical and numerical results.

[1] Michael Kunzinger, Hermann Schichl, Roland Steinbauer, and James A. Vickers. Global Gronwall estimates for integral curves on Riemannian manifolds. *Rev. Mat. Complut.*, 19(1):133–137, 2006.

[2] John W. Simpson-Porco and Francesco Bullo. Contraction theory on Riemannian manifolds, *Systems & Control Letters* 65 (2014), 74-80.

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