Half-explicit integrators for constrained mechanical systems on Lie groups

Martin Arnold
Martin Luther University Halle-Wittenberg, Germany
martin.arnold@mathematik.uni-halle.de

Configuration space with Lie group structure have become a quasi-standard in the dynamical simulation of flexible multibody systems with large rotations (O. Brüls, A. Cardona, M. Arnold: Lie group generalized-α time integration of constrained flexible multibody systems. Mechanism and Machine Theory 48:121-137, 2012). The equations of motion form constrained systems on Lie groups that are isomorphic to Cartesian products of (direct or semi-direct) products of $\mathbb{R}^3$ and SO(3). For these Lie groups, the exponential map exp, its right trivialized tangent $d\exp$ and the inverse $d\exp^{-1}$ may be evaluated in closed form with a computational complexity that compares to the classical Rodrigues formula for exaluating exp on SO(3). Therefore, local parametrizations in terms of elements of the corresponding Lie algebra may be used efficiently.

Combining the classical Munthe-Kaas approach for Runge-Kutta Lie group time integration and a half-explicit strategy for solving constrained systems (M. Arnold: Half-explicit Runge-Kutta methods with explicit stages for differential-algebraic systems of index 2 . BIT Numerical Mathematics 38:415-438, 1998), we end up with a half-explicit Lie group integrator for constrained mechanical systems. Up to order $p = 5$, methods with a reasonable number of half-explicit Runge-Kutta stages have been constructed including the HELieDOP5 integrator that generalizes the well known 5th order Dormand and Prince method to constrained mechanical systems on Lie groups. The results of the theoretical convergence analysis are verified by numerical tests for some classical benchmark problems.

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