Eulerian and Lagrangian stability in Zeitlin's model of hydrodynamics

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The 2-D Euler equations of a perfect fluid possess a beautiful geometric description: they are reduced geodesic equations on the infinite-dimensional Lie group of symplectomorphims with respect to a right-invariant Riemannian metric. This structure enables insights to Eulerian and Lagrangian stability via sectional curvature and Jacobi equations.

The Zeitlin model is a finite-dimensional analog of the 2-D Euler equations; the only known discretization that preserves the rich geometric structure. Theoretical and numerical studies indicate that Zeitlin's model provides consistent long-time behaviour on large scales, but to which extent it truly reflects the Euler equations is mainly open. Towards progress, I present here two new results. First, convergence of the sectional curvature in the Euler–Zeitlin equations on the Lie algebra $\mathfrak{su}(N)$ to that of the Euler equations on the sphere. Second, L^2-convergence of the corresponding Jacobi equations for Lagrangian and Eulerian stability. The new results allow geometric conclusions about Zeitlin's model to be transferred to Euler's equations and vice versa.

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