Learning linear operators: infinite-dimensional regression as an inverse problem

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We consider the problem of learning a linear operator between two Hilbert spaces from empirical observations, which we interpret as least squares regression in infinite dimensions. We show that this goal can be reformulated as a statistical inverse problem with unknown noncompact forward operator. However, we prove that, in terms of spectral properties, this problem is equivalent to the well-known compact inverse problem with scalar response regression. Our framework allows for the elegant derivation of dimension-free rates for generic learning algorithms under Hölder-type source conditions. The rates holds for a variety of relevant scenarios in functional regression and nonparametric regression with operator-valued kernels and match those of classical kernel regression with scalar response.

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