Modified equations and backward error analysis for stochastic Hamiltonian systems

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In this work, we address our attention on long-term analysis of numerical discretizations to stochastic Hamiltonian systems of Ito and Stratonovich types. Specifically, it is well-known that, in Ito scenario, the averaged Hamiltonian computed along the flow of such systems grows linearly in time; instead, for Stratonovich systems, the Hamiltonian remains pathwise constant. Hence, our aim is to detect how numerical Hamiltonians behave, in terms of being able to show the characteristic behavior of the exact flows along the numerical dynamics, over long time windows. Then, we perform a weak backward error analysis for such systems based on the construction of weak modified differential equations. These equations allow us to understand the trend of Hamiltonians along numerical trajectories. It turns out that, in general, extra error terms arise and, in particular, in the Stratonovich case, an exponential growth of the error is exhibited, after time intervals of length $O(\Delta t^{-p})$, being p the weak order of the analyzed method. Finally, selected numerical experiments are given to confirm the theoretical analysis. The results are based on a joint work with Raffaele D'Ambrosio (University of L'Aquila).

Joint work with Raffaele D'Ambrosio (University of L'Aquila).