The robustness of the Euler scheme for scalar SDEs with non-Lipschitz diffusion coefficient

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We consider stochastic differential equations (SDEs) that are given by

$$dV_t = a(V_t)dt + (b(V_t))^{1-\gamma}\, dW_t, \qquad t \in [0,T],$$

where $V_0 = v_0 \in \mathbf{R}$ is deterministic, $W = (W_t)_{t\in[0,T]}$ is a Brownian motion and $\gamma \in \left(0, \frac{1}{2}\right]$. We assume that $a : \mathbf{R} \to \mathbf{R}$ and $b : \mathbf{R} \to [0,\infty)$ are globally Lipschitz continuous. Well-known examples that fall into this class of SDEs are the CIR process, the CEV process or the Wright-Fisher diffusion.

We analyze the equidistant Euler scheme for the above SDE and, among other results, we show L^1-convergence order $1/2 - \varepsilon$ in the discretization points (for $\varepsilon > 0$ arbitrarily small) if

$$\int_0^T \mathbf{E}\left[\frac{1}{b(V_t)^{2\gamma}}\right] dt < \infty.$$

Thus, the loss of Lipschitzness, i.e. $\gamma > 0$, for the diffusion coefficient can be compensated by an appropriate inverse moment condition. This result yields in particular a unifying framework for the above mentioned SDEs: for the CIR or Wright-Fisher process, the above condition corresponds to the non-attainability of the boundaries of their support, while for the CEV process this inverse moment condition is always fulfilled.

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