Numerical analysis of Euler scheme for SDE driven by fractional Brownian motion

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In a first part, we will present how to use stochastic sewing lemma from [Le20] for building solutions of SDE driven by additive fractional Brownian motion. When the drift is regular or bounded, we can build these solutions as the limit of Euler schemes and obtain a strong rate of convergence, as in [BDG21,DGL21,DAGI19].

Moreover, using roughness of fractional Brownian motion, we can define solutions to SDEs with distributional drift as limit of the previous solutions build with bounded regular drifts. In a second part, we will present numerical simulations of these singular SDEs involving Dirac measure or indicator functions in dimension 1 or 2.

Finally, we will present how to obtain a strong rate of convergence by combining the speed of approximation of the distributional drift and the time-step size in the Euler scheme. However it will force the noise to be "rough enough", essentially by adding constraints on the Hölder regularity of the noise [GHR22].

References

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Joint work with El Mehdi Haress (MICS - CentraleSupélec - Paris-Saclay University, France) and Alexandre Richard (MICS - CentraleSupélec - Paris-Saclay University, France).