Learning High-Dimensional McKean-Vlasov Forward-Backward Stochastic Differential Equations with General Distribution Dependence

Jiequn Han
Flatiron Institute, Simons Foundation, USA
jhan@flatironinstitute.org

In this talk, we introduce a novel deep learning approach to solve McKean-Vlasov forward-backward stochastic differential equations (MV-FBSDEs), a core challenge in mean-field control and mean-field games. Our method overcomes limitations of existing techniques by addressing full distribution dependence in mean-field interactions.

By building on fictitious play, we transform the problem into repeatedly solving standard FBSDEs with explicit coefficient functions. These coefficient functions are used to approximate the MV-FBSDEs' model coefficients with full distribution dependence, and are updated by solving another supervising learning problem using training data simulated from the last iteration's FBSDE solutions. We use deep neural networks to solve standard BSDEs and approximate coefficient functions in order to solve high-dimensional MV-FBSDEs. Under suitable assumptions, we demonstrate convergence without the curse of dimensionality using integral probability metrics. We present the numerical performance in high-dimensional MV-FBSDE problems, including a mean-field game example of the well-known Cucker-Smale model whose cost depends on the full distribution of the forward process.

Joint work with Ruimeng Hu (University of California, Santa Barbara, USA) and Jihao Long (Princeton University, USA).