Behavior of solutions to the 1D focusing stochastic L^2-critical and supercritical nonlinear Schrödinger equation

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We study the focusing stochastic nonlinear Schrödinger equation in 1D in the L^2-critical and supercritical cases with an additive or multiplicative perturbation driven by space-time white or space colored noise. Unlike the deterministic case, the Hamiltonian (or energy) is not conserved in the stochastic setting, nor is the mass (i.e. L^2-norm) conserved in the additive case.

For a space-time white noise, we show that the noise may induce blow-up, thus, ceasing the global existence of the solution, which otherwise would be global in the deterministic setting. Furthermore, we study the effect of the noise on the blow-up dynamics in both multiplicative and additive noise settings. We obtain profiles and rates of the blow-up solutions.

For a space-correlated noise, we investigate both theoretically and numerically how the energy is affected by various types of space correlation, and its dependence on the discretization parameters and the schemes. We then perform numerical investigation of the noise influence on the global dynamics, measuring the probability of blow-up versus scattering behavior depending on parameters of correlation kernels. Finally, we study numerically the effect of the spatially correlated noise on the blow-up behavior.

We conclude that when blow-up occurs, such random perturbations do not influence the blow-up dynamics, except for shifting of the blow-up center location.

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