Solving the stochastic Helmholtz problem by the method of moment functions

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In this work, we study the solvability of the stochastic Helmholtz problem in the sense of the equivalence of equations in the mean and in the quadratic mean. We consider the linear statement of the problem. This is explained by the efficiency of the method of moment functions as applied to linear equations, whereas the efficiency of this method markedly decreases when applied to nonlinear equations. The essence of the method of moment functions is that it reduces the study of a stochastic equation to the study of a system of ordinary differential equations with respect to the moments under consideration.

The given the system of second-order stochastic Ito equations linear in drift

$$\ddot{x}_i = a_{ik}(t)\dot{x}_k + b_{ik}(t)x_k + \sigma_{ij}(t,x,\dot{x})\dot{\xi}^j, \quad (i=\overline{1,n}; j=\overline{1,m}).,\tag{1}$$

reduce it to equivalent equations of the Lagrangian structure. $\xi = (\xi_1(t,\omega),\ldots,\xi_r(t,\omega))^T$ is a system of random processes with independent increments.

To solve the problem, we apply the operation of mathematical expectation $M(\cdot)$ to Eq. (1). By introducing the denotation $m_\nu(t) = Mx_\nu(t)$, we obtain the equation

$$\ddot{m}_i = a_{ik}(t)\dot{m}_k + b_{ik}(t)m_k, \quad (i=\overline{1,n};\ k=\overline{1,n}).\tag{2}$$

We now formulate the indirect Helmholtz problem in the space $(m,\dot{m})$ as follows: given Eq. (2), determine the conditions for the functions h_i^ν and the Lagrange function $L = L(m,\dot{m},t)$, under which the following relations hold

$$h_i^\nu(\ddot{m}_i - a_{ik}(t)\dot{m}_k - b_{ik}(t)m_k) = \frac{d}{dt}(\frac{\partial L}{\partial \dot{m}_i}) - \frac{\partial L}{\partial m_i}.$$

We construct the equations of the Lagrangian structure by the given second-order linear stochastic Ito equations in the spaces of moment functions of both the first and second order. Necessary and sufficient conditions for the direct and indirect representation of the Lagrangian are obtained.

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