Averaging method and two-sided bounded solutions on the axis of systems with impulsive effects at non-fixed times

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The averaging method, originally offered by Krylov and Bogolyubov for ordinary differential equations, is one of the most widespread and effective methods for the analysis of nonlinear dynamical systems. Further, the averaging method were developed and applied for investigating of various problems. As well-known, impulsive systems of differential equations supply as mathematical models of objects that, in the course of their evolution, they are subjected to the action of short-term forces. Much research has been devoted to non-fixed impulse problems. For these problems, the existence, stability, and other asymptotic properties of solutions were studied and boundary value problems for impulsive systems were considered. Questions of the existence of periodic and almost periodic solutions of impulsive systems also are considered.

In this communication, the averaging method is used to study the existence of two-sided solutions bounding on the axis of impulse systems of differential equations with non-fixed times. It is shown that a one-sided, bounding, asymptotically stable solution to the averaged system generates a two-sided solution to the exact system. The closeness of the corresponding solutions of the exact and averaged systems both on finite and on infinite time intervals is substantiated by the first and second theorems of N.N. Bogolyubov.

In this paper, we consider a exact system of differential equations with impulsive effects at non-fixed times and a small parameter of the following form

$$\dot{x}(t) = \varepsilon X(t,x), \qquad t \neq t_i(x),$$

$$\Delta x\Big|_{t=t_i(x)} = \varepsilon I_i(x)$$

$$x(0) = x_0$$

where $\varepsilon > 0$ is a small parameter, $t_i(x) < t_{i+1}(x)$ $(i = 1, 2, \ldots)$ moments of impulsive effects, functions X and I_i d are n - dimensional vector of functions.

We put $U_a = \{x \in R^d : |x| \leq a\}$. Suppose the following conditions are met:

1. The functions $X(t,x)$ and $I_i(x)$ are continuous in the set $Q = \{t \geq 0, x \in U_a\}$, bounded by a constant $M > 0$, and in x satisfy the Lipschitz condition with a constant $L > 0$;

2. Uniformly in t, x for $t \geq 0, x \in U_a$, there exist finite limits

$$X_0(x) = \lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} X(s,x)ds,$$

$$I_0(x) = \lim_{T\to\infty} \frac{1}{T} \sum_{t<t_i(x)<T} I_i(x),$$

3. Solution $y = y(t), y(0) = x(0)$ of the averaged system

$$\dot{y} = \varepsilon[X_0(y) + I_0(y)]$$

is defined for $t \geq 0$ and lies in U_a together with some neighborhood ρ and is uniformly asymptotically stable;

4. The moments of the impulsive effect $t_i(x)$ are continuously and their functions satisfy in U_a uniformly in $i \in N$, and the surfaces $t = t_i(x)$ satisfy the separation condition, that is

$$\min_{x \in U_a} t_{i+1}(x) < \min_{x \in U_a} t_i(x) (i = 1, 2, \ldots)$$

Suppose that there is a constant $C > 0$ such that for all $t > 0$ and $x \in U_a$

$$i(t, x) \leq Ct$$

where $i(t, x)$ is the number of pulses on $(0, t)$.

It is also assumed that the solutions of exact system intersect each surface $t = t_i(x)$ at most once, that is, there is no beating. The conditions for the absence of beating are well studied.

Theorem 1. Let Conditions 1-4 be satisfied. Then, for an arbitrary $\eta > 0$, one can specify ε_0 such that $\varepsilon < \varepsilon_0$ for $t \geq 0$, the inequality

$$|x(t) - y(t)| < \eta$$

where $x(t)(x(0) = y(0) = x_0)$ is a solution to the exact system .

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