Rigorous numerics, Poincaré maps, and Covering Relations in infinite-dimensional spaces for computer assisted proofs in Delay Differential Equations.

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We will show some general principles of a recently developed high-order Lohner-type algorithm for a rigorous integration of systems of Delay Differential Equations (DDEs) [1], together with some topological tools in the infinite dimensional phase-space of DDEs that are suitable for computer assisted proofs.

We will use these tools to prove, with the computer assistance, various kinds of dynamical behaviour, for example, existence of several (apparently) unstable periodic orbits in Mackey-Glass Equation (in the regime of classical values of parameters where chaos is numerically observed), persistence of symbolic dynamics in a delay-perturbed chaotic ODE (the Rössler system), and the rigorous computation of trajectories in piecewise defined DDEs.

The method is quite general and does not impose severe restrictions on the kind of solutions it can track, i.e. the integration time does not need to be a multiple of the basic time lag nor the solutions need not to be of a specific class, e.g. periodic.

[1] R. Szczelina and P. Zgliczyński. High-order Lohner-type algorithm for rigorous computation of Poincaré maps in systems of Delay Differential Equations with several delays, *Accepted for publication in Found. Comput. Math., preprint: https://arxiv.org/abs/2206.13873*, (2023).

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