Validated integration of semilinear parabolic PDEs

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Simulations are at the core of scientific computing, but their mathematical reliability is often difficult to quantify, especially when one is interested in the output of a given simulation, rather than in the asymptotic regime where the discretization parameter tends to zero. In this talk, we present a computer-assisted proof methodology to perform rigorous time integration for semilinear parabolic PDEs with periodic boundary conditions. We formulate an equivalent zero-finding problem based on a variations of constants formula in Fourier space. Using Chebyshev interpolation and domain decomposition in time, we then finish the proof with a Newton-Kantorovich type argument. The final output of this procedure is a proof of existence of an orbit, together with guaranteed error bounds between this orbit and a numerically computed approximation. We illustrate the versatility of the approach with results for the Fisher equation, the Swift-Hohenberg equation, the Ohta-Kawasaki equation and the Kuramoto–Sivashinsky equation. We expect that this rigorous integrator can form the basis for studying boundary value problems for connecting orbits in partial differential equations.

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