Computational method for solving a boundary value problem for an impulsive integro-differential equation

Sandugash Mynbayeva
K.Zhubanov Aktobe Regional University, Kazakhstan
mynbaevast80@gmail.com

In this study, the boundary value problem for the Fredholm integro-differential equation subject to impulse effects at fixed time points is considered:

$$\frac{dy}{dt} = A(t)y + \sum_{l=1}^{k} \int_0^T \varphi_l(t)\psi_l(s)y(s)ds + f(t),\ t \in (0,T) \setminus \{\theta_1, \theta_2, \dots, \theta_m\},\ y \in R^n, \tag{1}$$

$$(0 = \theta_0 < \theta_1 < \ldots < \theta_m < \theta_{m+1} = T),$$

$$By(0) + Cy(T) = d, \quad d \in R^n, \tag{2}$$

$$\Delta y(\theta_j) = \sum_{i=0}^{j-1} d_{ij} y(\theta_i + 0), \quad j = \overline{1,m}, \tag{3}$$

where $\Delta y(\theta_j) = y(\theta_j + 0) - y(\theta_j - 0)$, the square matrices $A(t)$, $\varphi_l(t)$, and $\psi_l(s)$ of order n are continuous on $[0,T]$, $f(t)$ is piecewise continuous on $[0,T]$, with the possible exception of the points $t = \theta_j, j = \overline{1,m}$, the square matrices B, C, and d_{ij} of order n and the vector d are constant.

A constructive method for solving the problem based on the Dzhumabaev para- metrization method is proposed, and an algorithm for finding a numerical solution to the problem is proposed. The algorithms consist of constructing and solving the systems of linear algebraic equations with respect to the arbitrary parameters. Since the Cauchy problems are solved independently from each other, their solutions can be found by parallel computing. In our calculations, we used the fourth-order Runge-Kutta method, the Adams and Bulirsch-Stoer methods for solving auxiliary Cauchy problems. We also used used Simpson's rule for estimating definite integrals.

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