Numerical solution of a semi-periodic initial problem of the fourth order hyperbolic type using a generalized operator and functional parametrization

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The domain $\Omega = [0,T] \times [0,\omega]$ we consider the following semi-periodic initial boundary value problem for a fourth order system of partial differential equations:

$$\frac{\partial^4 u}{\partial t^3 \partial x} = A_1(t,x)\frac{\partial^3 u}{\partial t^2 \partial x} + A_2(t,x)\frac{\partial^3 u}{\partial t^3} + A_3(t,x)\frac{\partial^2 u}{\partial t^2} + A_4(t,x)\frac{\partial^2 u}{\partial t \partial x} +$$

$$+A_5(t,x)\frac{\partial u}{\partial t} + A_6(t,x)\frac{\partial u}{\partial x} + A_7(t,x)u + f(t,x),$$

$$u(0,x) = \varphi_1(x), \quad x \in [0,\omega],$$

$$\frac{\partial u(t,x)}{\partial t}\Big|_{t=0} = \varphi_2(x), \quad x \in [0,\omega],)$$

$$\frac{\partial^2 u(t,x)}{\partial^2 t}\Big|_{t=0} = \frac{\partial^2 u(t,x)}{\partial^2 t}\Big|_{t=T}, \quad x \in [0,\omega],$$

$$u(t,0) = \psi(x), \quad t \in [0,T],$$

where $u(t,x) = col(u_1(t,x), u_2(t,x), \ldots, u_n(t,x))$ is the unknown function; the $n \times n$-matrices $A_i(t,x), (i = \overline{1,7})$, and n- vector function $f(t,x)$ are continuous on Ω; n vector-function $\psi(t)$ are continuously three times differentiable on $[0,T]$; the n vector-functions $\varphi_1(x)$ and $\varphi_2(x)$ are continuously differentiable on $[0,\omega]$.

Reducing the order of the problem by two times, we reduce it to the Cauchy problem for a system of n first-order ordinary differential equations. It is shown that the Cauchy problem has a solution using the method of generalized operations or the method of functional parameterization. Using numerical methods for one variable of the Cauchy problem, solutions of the Cauchy problem are obtained.

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