Semidefinite games

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We introduce and study the class of semidefinite games, which generalizes bimatrix games and finite N-person games, by replacing the simplex of the mixed strategies for each player by a slice of the positive semidefinite cone in the space of real symmetric matrices.

For semidefinite two-player zero-sum games, we show that the optimal strategies can be computed by semidefinite programming. Furthermore, we show that two-player semidefinite zero-sum games are almost equivalent to semidefinite programming, generalizing Dantzig's result on the almost equivalence of bimatrix games and linear programming.

For general two-player semidefinite games, we prove a spectrahedral characterization of the Nash equilibria. Moreover, we give constructions of semidefinite games with many Nash equilibria. In particular, we give a construction of semidefinite games whose number of connected components of Nash equilibria exceeds the long standing best known construction for many Nash equilibria in bimatrix games, which was presented by von Stengel in 1999.

Joint work with Thorsten Theobald (Goethe Universität Frankfurt am Main, Germany) and Elias Tsigaridas (Sorbonne Université, Paris University, CNRS, and Inria Paris, France).