Novel algorithms for nonconvex second-order optimization with strong performance guarantees

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Second-order optimization has recently experienced significant developments, leading to numerous fruitful applications in science and engineering. In particular, recent research has shown that a second-order stationary point (SOSP) of a nonconvex optimization problem is often a globally optimal solution for instances that arise in areas such as machine learning and statistics. Therefore, developing efficient algorithms for computing such points is pivotal for advancing those areas. Our research introduces new algorithms with substantial theoretical improvements for solving two types of nonconvex constrained optimization problems and conducts numerical studies to show the practical advantages of the proposed methods over the state-of-the-art methods. In our first work, we develop a novel augmented Lagrangian (AL) method for finding an SOSP of a nonconvex equality constrained optimization problem. Theoretically, the proposed AL method improves upon the best-known complexity guarantees. Numerically, the computational speed of our AL method is vastly faster than the competing ones for solving a classical statistical problem. In our second work, we design efficient algorithms to find an SOSP of a general conic constrained nonconvex optimization problem. For the first time, we introduce a notion of an SOSP for general conic constrained optimization and propose an efficient algorithm to find it. Numerical results demonstrate the significant potential superiority of our barrier-AL method over the state-of-the-art method in terms of solution quality.

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