Adaptive regularization without function values

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An adaptive regularization algorithm for unconstrained nonconvex optimization is presented in which the objective function is never evaluated, but only derivatives are used. This algorithm belongs to the class of adaptive regularization methods, for which optimal worst-case complexity results are known for the standard framework where the objective function is evaluated. It is shown in this paper that these excellent complexity bounds are also valid for the new algorithm, despite the fact that significantly less information is used. In particular, it is shown that, if derivatives of degree one to p are used, the algorithm will find an ϵ_1-approximate first-order minimizer in at most $(\epsilon_1^{-(p+1)/p})$ iterations, and an (ϵ_1, ϵ_2)-approximate second-order minimizer in at most $(\max(\epsilon_1^{-(p+1)/p}, \epsilon_2^{-(p+1)/(p-1)}))$ iterations. As a special case, the new algorithm using first and second derivatives, when applied to functions with Lipschitz continuous Hessian, will find an iterate x_k at which the gradient's norm is less than ϵ_1 in at most $(\epsilon_1^{-3/2})$ iterations.

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