Minimal Euclidean Distance degree of Segre-Veronese varieties

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The Euclidean Distance degree $\mathrm{EDdeg}_Q(X)$ of an algebraic variety X in an inner product space $(\mathbb{R}^N, Q)$ counts the number of complex critical points of the distance function $x \in X \mapsto Q(v-x, v-x)$ from a generic point $v \in \mathbb{R}^N$ to X. Since this invariant of X depends on Q, it is a natural problem to find or characterize inner products Q that correspond to the minimal possible $\mathrm{EDdeg}_Q(X)$. In my talk I will discuss this question for Segre-Veronese varieties, which consist of rank-1 (partially symmetric) tensors. I will show that with respect to the classical Frobenius product $F(A,B) = \sum_{i=1}^n \sum_{j=1}^m A_{ij}B_{ij}$, the variety X of $n \times m$ rank-1 matrices has smallest $\mathrm{EDdeg}_F(X) = \min(n,m)$, whereas $\mathrm{EDdeg}_Q(X)$ with respect to a sufficiently general inner product Q on $\mathbb{R}^N = \mathbb{R}^{n\times m}$ is much higher.

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