WHAT IS THE PROBABILITY THAT A RANDOM SYMMETRIC TENSOR IS CLOSE TO RANK-ONE?

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We address the general problem of estimating the probability that a real symmetric tensor is close to rank-one tensors. Using Weyl's tube formula, we turn this question into a differential geometric one involving the study of metric invariants of the real Veronese variety. More precisely, we give an explicit formula for its reach and curvature coefficients with respect to the Bombieri-Weyl metric.

These results are obtained using techniques from Random Matrix theory and an explicit description of the second fundamental form of the Veronese variety in terms of GOE matrices. Our findings give a complete solution to the original problem, and in the case of rational normal curves lead to some novel asymptotic results.

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