Rigid continuation paths I. Quasilinear average complexity for solving polynomial systems

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How many operations do we need on average to compute an approximate root of a random Gaussian polynomial system? Beyond Smale's 17th problem that asked whether a polynomial bound is possible, we prove a quasi-optimal bound $(\text{input size})^{1+o(1)}$. This improves upon the previously known $(\text{input size})^{\frac{3}{2}+o(1)}$ bound.

The new algorithm relies on numerical continuation along rigid continuation paths. The central idea is to consider rigid motions of the equations rather than line segments in the linear space of all polynomial systems. This leads to a better average condition number and allows for bigger steps. We show that on average, we can compute one approximate root of a random Gaussian polynomial system of n equations of degree at most D in $n+1$ homogeneous variables with $O(n^4D^2)$ continuation steps. This is a significant improvement over previous bounds that prove no better than $\sqrt{2}^{\min(n,D)}$ continuation steps on average.

This talk will also be an introduction to Felipe Cucker's talk in the same session.