Some theoretical properties of physics-informed neural networks

Nathan Doumèche
Sorbonne Université, France
nathan.doumeche@sorbonne-universite.fr

Physics-informed neural networks (PINNs) combine the expressiveness of neural networks with the interpretability of physical modeling. Their good practical performance has been demonstrated both in the context of solving partial differential equations and in the context of hybrid modeling, which consists of combining an imperfect physical model with noisy observations. However, most of their theoretical properties remain to be established. We show that classical training of these networks suffers from systematic overfitting. We then show that adding a ridge regularization to their empirical risk ensures the consistency of the resulting estimator. However, the convergence of PINNs to a solution that verifies the regularized physical constraint requires a more involved analysis , which mobilizes tools from functional analysis.

Joint work with Gérard Biau (Sorbonne Université) and Claire Boyer (Sorbonne Université).