# Pseudospectral Shattering for Generalized Eigenvalues and Inverse-Free Matrix Pencil Diagonalization

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We present a randomized, inverse-free algorithm for producing an approximate diagonalization of any $n \times n$ matrix pencil $(A, B)$. The bulk of the algorithm rests on a randomized divide-and-conquer eigensolver for the generalized eigenvalue problem originally proposed by Ballard, Demmel, and Dumitriu [Technical Report 2010]. We demonstrate that this divide-and-conquer approach can be formulated to succeed with high probability as long as the input pencil is sufficiently well-behaved. This is accomplished by generalizing the recent pseudospectral shattering work of Banks, Garza-Vargas, Kulkarni, and Srivastava [Foundations of Computational Mathematics 2022]. The resulting randomized algorithm (with high probability and in exact arithmetic) produces invertible $S, T$ and diagonal $D$ such that $||A - SDT^{-1}||_2 \leq \varepsilon$ and $||B - SI_nT^{-1}||_2 \leq \varepsilon$ in at most $O\left(\log(n) \log^2\left(\frac{n}{\varepsilon}\right) T_{\mathrm{MM}}(n)\right)$ operations, where $T_{\mathrm{MM}}(n)$ is the complexity of $n \times n$ matrix multiplication. This not only provides a new set of guarantees for highly parallel generalized eigenvalue solvers but also establishes nearly matrix multiplication time as an upper bound on the complexity of exact arithmetic matrix pencil diagonalization.

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