Extreme singular values of inhomogeneous, sparse, rectangular random matrices

Ioana Dumitriu
University of California, San Diego, USA
idumitriu@ucsd.edu

We develop a unified approach to bounding the largest and smallest singular values of an inhomogeneous random rectangular matrix, based on the non-backtracking operator and the Ihara-Bass formula for general Hermitian matrices with a bipartite block structure. Our main results are probabilistic upper/lower bounds for the largest/smallest singular values of a large rectangular random matrix X. These bounds are given in terms of the maximal and minimal $\updownarrow_2$-norms of the rows and columns of the variance profile of X. The proofs involve finding probabilistic upper bounds on the spectral radius of an associated non-backtracking matrix B.

The two-sided bounds can be applied to the centered adjacency matrix of sparse inhomogeneous Erdos-Renyi bipartite graphs for a wide range of sparsity. In particular, for Erdos-Renyi bipartite graphs $G(n, m, p)$ with $p = \omega(logn)/n$, and $m/n \to y \in (0, 1)$, our sharp bounds imply that there are no outliers outside the support of the Marcenko-Pastur law almost surely.

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