Preserving and Exploiting Symmetry in Multivariate interpolation

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Interpolation is a prime tool in algebraic computation while symmetry is a qualitative feature that can be more relevant to a mathematical model than the numerical accuracy of the parameters. We shall show how to exactly preserve symmetry in multivariate interpolation while exploiting it to alleviate the computational cost. We revisit minimal degree and least interpolation with symmetry adapted bases, rather than monomial bases.

An interpolation problem is defined by a set of linear forms on the polynomial ring and values to be achieved by an interpolant. For Lagrange interpolation the linear forms consist of evaluations at some nodes, while Hermite interpolation also considers the values of successive derivatives. Both are examples of ideal interpolation in that the kernels of the linear forms intersect into an ideal.

For a space of linear forms invariant under a group action, we construct bases of invariant interpolation spaces in blocks, capturing the inherent redundancy in the computations. With the so constructed symmetry adapted interpolation bases, the uniquely defined interpolant automatically preserves any equivariance the interpolation problem might have. Even with no equivariance, the computational cost to obtain the interpolant is alleviated thanks to the smaller size of the matrices to be inverted.

For an ideal interpolation problem with symmetry, we address the simultaneous computation of a symmetry adapted basis of the least interpolation space and the symmetry adapted H-basis of the ideal. Beside its manifest presence in the output, symmetry is exploited computationally at all stages of the algorithm. For an ideal invariant, under a group action, defined by a Groebner basis, the algorithm allows to obtain a symmetry adapted basis of the quotient and of the generators. We shall also note how it applies surprisingly but straightforwardly to compute fundamental invariants and equivariants of a reflection group.

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