MINIMIZATION OF ANALYTIC FUNCTIONS OVER COMPACT DOMAINS

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In this talk, we propose a new method to minimize analytic functions over compact domains through the use of polynomial approximations. This is in essence an effective application of the Stone-Weierstrass Theorem, as we seek to construct a polynomial approximant of f over a compact domain satisfying an arbitrary set precision. The polynomial approximation allows us to compute all critical points of the approximant exactly, using methods from computer algebra. Our Main Theorem provides conditions of probabilistic nature on the local minima of the objective function and on the accuracy of the polynomial approximation sufficient to guarantee that all local minima located in the interior of the compact domain are captured by the critical points of the polynomial. We provide an implementation of a probabilistic method to construct a polynomial least squares approximant of low degree, compute its critical points and initialize local minimization methods on the objective function f at these points, in order to recover the totality of its local minima .

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