Advances in phaseless sampling of the short-time Fourier transform

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Short-time Fourier transform (STFT) phase retrieval refers to the reconstruction of a function $f \in L^2(\mathbb{R})$ from phaseless samples of the form

$$|V_g f(\Lambda)| := \{|V_g f(\lambda)| : \lambda \in \Lambda\},$$

where $V_g f$ denotes the STFT of f with respect to a window function $g \in L^2(\mathbb{R})$, and $\Lambda \subseteq \mathbb{R}^2$ is a set of sampling locations. We present recent advances in STFT phase retrieval and focus on the question for which window functions g and which sets Λ is every f uniquely determined (up to a global phase) by the samples $|V_g f(\Lambda)|$. It turns out, that the phaseless sampling problem differs from ordinary sampling in a rather fundamental way: if $\Lambda = A\mathbb{Z}^2$ is a lattice then uniqueness is unachievable, independent of the choice of the window function and the density of the lattice. On the basis of this discretization barrier, we present possible ways to overcome it. We show that a restriction of the function class from $L^2(\mathbb{R})$ to certain subspaces of $L^2(\mathbb{R})$ yields uniqueness of the problem if one samples on sufficiently dense lattices. Finally, we highlight that without any restriction of the function class, a discretization is still possible: either one changes the sampling scheme from sampling on ordinary lattices to so-called square-root lattices, or one considers a multi-window system with 4 window functions . These results constitute the first general uniqueness theorems for the STFT phase retrieval problem.

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