Data-driven regularization for inverse problems - the dos and don‘ts

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Abstract: Inverse problems are about the reconstruction of an unknown physical quantity from indirect measurements. They appear in a variety of places, from medical imaging, for instance MRI or CT, to remote sensing, for instance Radar, to material sciences and molecular biology, for instance electron microscopy. Here, inverse problems is a tool for looking inside specimen, resolving structures beyond the scale visible to the naked eye, and to quantify them. It is a mean for diagnosis, prediction and discovery. Most inverse problems of interest are ill-posed and require appropriate mathematical treatment for recovering meaningful solutions. Classically, such approaches are derived almost conclusively in a knowledge driven manner, constituting handcrafted mathematical models. Examples include variational regularization methods with Tikhonov regularization, the total variation and several sparsity-promoting regularizers such as the L1 norm of Wavelet coefficients of the solution. While such handcrafted approaches deliver mathematically rigorous and computationally robust solutions to inverse problems, they are also limited by our ability to model solution properties accurately and to realise these approaches in a computationally efficient manner. Recently, a new paradigm has been introduced to the regularization of inverse problems, which derives solutions to inverse problems in a data driven way. Here, the inversion approach is not mathematically modelled in the classical sense, but modelled by highly over-parametrised models, typically deep neural networks, that are adapted to the inverse problems at hand by appropriately selected training data. Current approaches that follow this new paradigm distinguish themselves through solution accuracies paired with computational efficieny that were previously unconceivable. In this lecture I will give an introduction to this new data-driven paradigm for inverse problems. Presented methods include data-driven variational models and plug-and-play approaches, learned iterative schemes aka learned unrolling, and learned post-processing. Throughout presenting these methodologies, we will discuss their theoretical properties and provide numerical examples for image denoising, deconvolution and computed tomography reconstruction. The lecture will finish with a discussion of open problems and future perspectives.