The limit empirical spectral distribution of random matrix polynomials

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Three famous classic results concern the distributions of the roots of a random polynomial and the eigenvalues of a random matrix or pencil. Under relatively mild assumptions on the distribution of the coefficients, the former is known to converge to the uniform distribution on the unit circle when the degree k approaches infinity. Under similarly unrestrictive assumptions on the distributions of the entries, the latter is known to converge to the uniform distribution on the unit disk (when the entries have mean 0 and variance $1/n$) when the size n approaches infinity. Several mathematicians have also independently derived the distribution of the generalised eigenvalues of a random pencil: in this case, the distribution is uniform on the Riemann sphere. However, until the present work nothing was to my knowledge known about the eigenvalues of a general random matrix polynomial, which could be thought as an intermediate case between a random matrix or pencil ($k = 1$) and a random polynomial ($n = 1$).

In this talk I plan to first give some gentle introduction, thought for non-experts on random variables and random matrices, to the known results mentioned above. I will then move on to describe recent new results that we obtained about the limit spectral distributions of a random matrix polynomial, both in the regime $k \to \infty$ and in the case $n \to \infty$. After discussing the (easier) nonmonic case, I will also comment on what changes if the random matrix polynomial is assumed to be monic, i.e., having all random coefficients except the leading one which is taken to be the identity matrix.

The main tools from our results come both from random matrix theory and from deterministic matrix theory. In particular, we exploit (1) the replacement principle, proven by Tao, Vu and Krishnapur, (2) the logarithmic potential approach as proposed by Girko, and (3) classic perturbation theory results.

Joint work with Giovanni Barbarino.