Solving the Parametric Eigenvalue Problem by Taylor Series and Chebyshev Expansion

Thomas Mach
University of Potsdam, Germany
mach@uni-potsdam.de

We discuss two approaches to solving the parametric (or stochastic) eigenvalue problem. One of them uses a Taylor expansion and the other a Chebyshev expansion. The parametric eigenvalue problem assumes that the matrix A depends on a parameter μ, where μ might be a random variable. Consequently, the eigenvalues and eigenvectors are also functions of μ. We compute a Taylor approximation of these functions about μ_0 by iteratively computing the Taylor coefficients. The complexity of this approach is $O(n^3)$ for all eigenpairs, if the derivatives of $A(\mu)$ at μ_0 are given. The Chebyshev expansion works similarly. We first find an initial approximation iteratively which we then refine with Newton's method. This second method is more expensive but provides a good approximation over the whole interval of the expansion instead around a single point.

We present numerical experiments confirming the complexity and demonstrating that the approaches are capable of tracking eigenvalues at intersection points. Further experiments shed light on the limitations of the Taylor expansion approach with respect to the distance from the expansion point μ_0.

This presentation is based on a paper available on arXiv, https://arxiv.org/abs/2302.03661, that has been submitted for publication.

Joint work with Melina A. Freitag (University of Potsdam, Germany).