# Universal Matrix Sparsifiers and Fast Deterministic Algorithms for Singular Value Approximation

**Cameron Musco**
University of Massachusetts Amherst, USA
cmusco@cs.umass.edu

Given a matrix $A \in \mathbb{R}^{n \times n}$ which is normalized so that its entries are bounded in magnitude by 1, it is well-known that randomly sparsifying $A$ to be supported on a subset of just $\tilde{O}(n/\epsilon^2)$ of its entries, yields an approximation $A_S$ with $\|A - A_S\|_2 \leq \epsilon n$ with high probability, where $\|\cdot\|_2$ is the spectral norm. We show that for positive semidefinite (PSD) matrices, no randomness is needed at all in this statement. Namely, there exists a fixed subset of $\tilde{O}(n/\epsilon^2)$ entries that acts as a *universal sparsifier*: $\|A - A_S\|_2 \leq \epsilon n$ holds simultaneously for every bounded entry PSD matrix. One can view this result as a significant extension of a spectral graph expander nearly matching the Ramanujan bound.

We leverage the existence of such universal sparsifiers to give the first *deterministic algorithms* for several central linear algebraic problems, including singular value and singular vector approximation and positive semidefiniteness testing, that run in faster than matrix multiplication time on worst-case input matrices. This partially addresses a significant gap between randomized and deterministic algorithms for fast linear algebraic computation. We also extend our results to non-PSD matrices and to deterministic algorithms that approximate $A$ with a non-sparse matrix. We prove lower bounds showing that many of our results are nearly optimal.

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