Avoiding discretization issues for nonlinear eigenvalue problems

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The first step when solving an infinite-dimensional eigenvalue problem is often to discretize it. We show that one must be extremely careful when discretizing nonlinear eigenvalue problems. Using examples, we show that discretization can: (1) introduce spurious eigenvalues, (2) entirely miss spectra, and (3) bring in severe ill-conditioning. While there are many eigensolvers for solving matrix nonlinear eigenvalue problems, we propose a solver for general holomorphic infinite-dimensional nonlinear eigenvalue problems that avoids discretization issues, which we prove is stable and converges. Moreover, we provide an algorithm that computes the problem's pseudospectra with explicit error control, allowing verification of computed spectra. The algorithm and numerical examples are publicly available in infNEP, which is a software package written in MATLAB. Finally, we show how this work fits into the bigger picture of recent progress in the foundations of infinite-dimensional computations.

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