

FAST COMPUTATIONS WITH ARROWHEAD AND DIAGONAL-PLUS-RANK-K MATRICES OVER
ASSOCIATIVE FIELDS

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We consider arrowhead and diagonal-plus-rank- k matrices (DPRk) in $\mathbb{F}^{n \times n}$ where $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}\}$, and \mathbb{H} is a non-commutative field of quaternions. Such matrices arise in many applications and computations with such matrices are parts of important linear algebra algorithms. We give formulas for matrix-vector multiplications, determinants, and inverses for both types of matrices. The formulas are unified in the sense that the same formulas hold in both, commutative and noncommutative associative algebras. Each formula requires $O(n)$ arithmetic operations. Most of the formulas hold for block matrices, as well. Further, we derived efficient $O(n^2)$ eigensolvers for quaternionic arrowhead and DPRk matrices. The eigensolvers use a version of Wielandt deflation technique. All algorithms are elegantly implemented using the polymorphism feature of the modern high-level computing language Julia. The code is available at <https://github.com/ivanslapnicar/MANAA>. Our results complement and extend the existing results for commutative fields.

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