Space-time adaptive low-rank approximation for high-dimensional parabolic PDEs

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In this talk we present the construction and analysis of a space-time adaptive method for parabolic partial differential equations that combines sparse wavelet expansions in time with adaptive low-rank approximations in the spatial variables. Similar to the existing adaptive low-rank method for elliptic problems, the method is based on a perturbed Richardson iteration, in the present case applied to a standard space-time variational formulation of the parabolic initial-boundary value problem. The analysis of the method requires a new approximation class for the temporal operator, taking into account the interaction between hierarchical tensor formats of different time indices. Since the parabolic operator is an isomorphism with respect to spaces not endowed with a cross norm, we devise a new low-rank preconditioning scheme based on exponential sum approximations that is adapted to the parabolic case. The method is shown to converge and satisfy similar complexity bounds as the existing adaptive low-rank methods for elliptic problems, establishing its suitability for parabolic problems on high-dimensional spatial domains. The construction also yields computable rigorous a posteriori error bounds for the total error depending on the activated basis functions and ranks in the approximation.

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